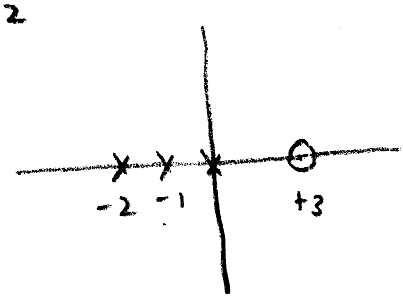
Signals & Systems Homework Solutions

S16)

a) $$X(s) = \frac{s-3}{s^3+3s^2+2s} = \frac{s-3}{s(s^2+3s+2)} = \frac{(s-3)}{s(s+1)(s+2)}$$

Partial Fraction expansion

$$X(s) = \frac{(s-3)}{s(s+1)(s+2)} = \frac{k_1}{s} + \frac{k_2}{s+1} + \frac{k_3}{s+2}$$



Cover Up

$$k_1 = \left.\frac{(s-3)}{(s+1)(s+2)}\right|_{s=0} = \frac{-3}{2}$$

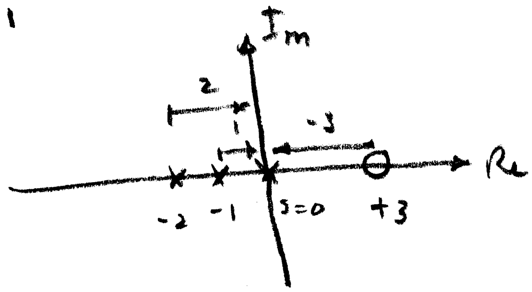
$$k_2 = \left.\frac{(s-3)}{s(s+2)}\right|_{s=-1} = \frac{(-1-3)}{(-1)(-1+2)} = \frac{-4}{(-1)(1)} = 4$$

$$k_3 = \left.\frac{(s-3)}{s(s+1)}\right|_{s=-2} = \frac{(-2-3)}{(-2)(-2+1)} = \frac{-5}{(-2)(-1)} = -\frac{5}{2}$$

$$\therefore \boxed{x(t) = -\frac{3}{2} + 4e^{-t} - \frac{5}{2}e^{-2t}}$$

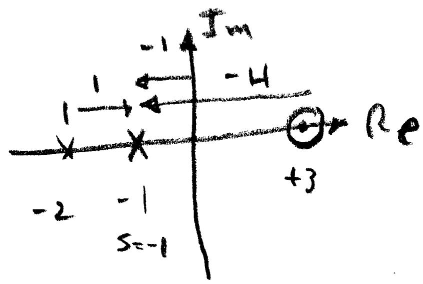
Graphically

for k_1
$s=0$



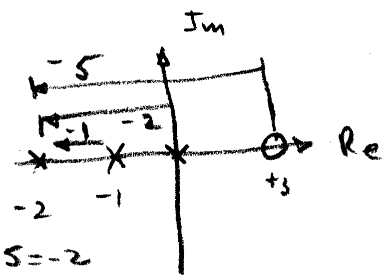
$$k_1 = \frac{-3}{(1)(2)} = -\frac{3}{2}$$

for k_2
$s=-1$



$$k_2 = \frac{-4}{(-1)(1)} = 4$$

for k_3
$s=-2$



$$k_3 = \frac{-5}{(-2)(-1)} = -\frac{5}{2}$$

b)

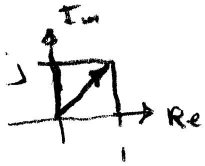
$$X(s) = \frac{s+1}{(s^2+1)} = \frac{s+1}{(s+j)(s-j)} = \frac{k_1}{(s+j)} + \frac{k_2}{(s-j)}$$

Coverup

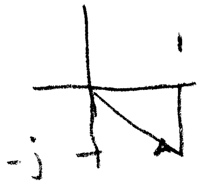
$$k_1 = \left.\frac{(s+1)}{(s-j)}\right|_{s=-j} = \frac{-j+1}{-2j} = \frac{1+j}{2}$$

$$k_2 = \left.\frac{(s+1)}{(s+j)}\right|_{s=j} = \frac{j+1}{2j} = \frac{1-j}{2} = k_1^*$$

$$X(s) = \frac{1+j}{2(s+j)} + \frac{(1-j)}{2(s-j)}$$



$1+j = \sqrt{2}\,e^{\frac{\pi}{4}j} \Rightarrow k_1 = \frac{1+j}{2} = \frac{1}{\sqrt{2}}e^{\frac{\pi}{4}j}$



$1-j = \sqrt{2}\,e^{-\frac{\pi}{4}j} \Rightarrow k_2 = \frac{1-j}{2} = \frac{1}{\sqrt{2}}e^{-\frac{\pi}{4}j}$

$$X(s) = \frac{1}{\sqrt{2}}\left(\frac{e^{\frac{\pi}{4}j}}{(s+j)} + \frac{e^{-\frac{\pi}{4}j}}{s-j}\right)$$

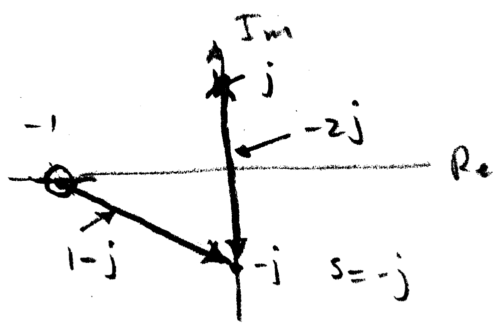
$$x(t) = \frac{1}{\sqrt{2}}\left(e^{-jt}\cdot e^{\frac{\pi}{4}j} + e^{+jt}e^{-\frac{\pi}{4}j}\right) = \frac{1}{\sqrt{2}}\left(e^{-j(t-\frac{\pi}{4})} + e^{+j(t-\frac{\pi}{4})}\right)$$

$$= \sqrt{2}\cos\left(t-\frac{\pi}{4}\right) = \sqrt{2}\left(\cos t\cos\frac{\pi}{4} + \sin t\sin\frac{\pi}{4}\right)$$

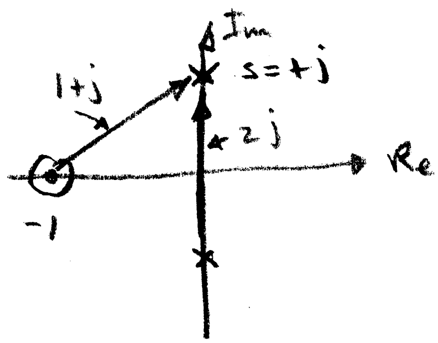
$$\boxed{x(t) = \cos t + \sin t}$$

Graphically.

for k_1
$s=-j$



for k_2
$s=j$



$k_2 = \frac{1+j}{2j} = \frac{j-1}{-2} = \frac{1-j}{2}$

c) $$X(s) = \frac{s+1}{s^2} = \frac{k_1}{s} + \frac{k_2}{s^2}$$

$$k_2 = (s+1)\Big|_{s=0} = 1$$

Substituting for k_2

$$\frac{(s+1)}{s^2} = \frac{k_1}{s} + \frac{1}{s^2}$$

or

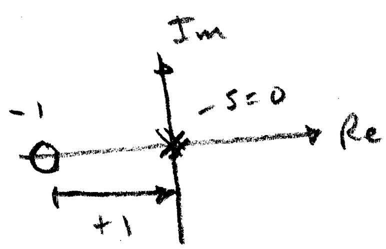
$$\frac{s}{s^2} = \frac{k_1}{s}$$

so $k_1 = 1$

$$X(s) = \frac{1}{s} + \frac{1}{s^2}$$

$$\boxed{x(t) = 1 + t}$$

We can do k_2 graphically



$k_2 = +1$

d) $X(s) = \dfrac{s+2}{s^2-1} = \dfrac{s+2}{(s+1)(s-1)}$

$$\frac{s+2}{(s+1)(s-1)} = \frac{k_1}{s+1} + \frac{k_2}{s-1}$$

<u>Cover Up</u>

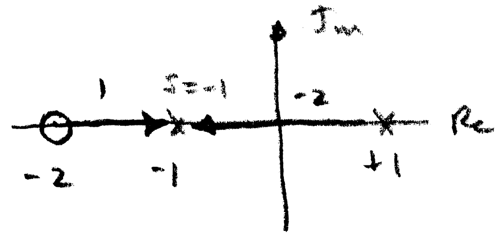
$$k_1 = \left.\frac{(s+2)}{s-1}\right|_{s=-1} = \frac{(-1+2)}{(-1-1)} = -\frac{1}{2}$$

$$k_2 = \left.\frac{(s+2)}{s+1}\right|_{s=+1} = \frac{(1+2)}{(1+1)} = \frac{3}{2}$$

$$\boxed{x(t) = \frac{3}{2}e^{t} - \frac{1}{2}e^{-t}}$$

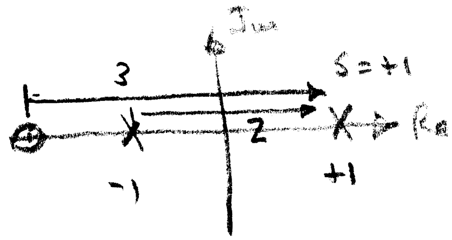
<u>Graphically</u>

for k_1
$s=-1$



$k_1 = \dfrac{1}{-2} = -\dfrac{1}{2}$

for k_2
$s=+1$



$k_2 = \dfrac{3}{2}$